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ON LEGENDRE ABSOLUTE DIFFERENCE CORDIAL LABELING OF SOME SPECIAL GRAPHS

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Abstract: **Cahit originally proposed the premise of cordial labelling in 1987. Legendre Cordial Labelling initially emerged by Jason D. Andoyo & F. Rulete in 2025. Inspired by this effort, we designed Legendre Absolute Difference Cordial Labelling. A bijection $\varrho: V(G) \to \{1, 2, .., |V|\}$ is typically referred to as a Legendre absolute difference cordial labelling modulo ρ, wherein ρ constitutes an odd prime, assuming G as basic connected graph while the resultant map $\varrho_\rho^*: E(G) \to \{0, 1\}$ laid out as $\varrho_\rho^*(\mu\nu) = 0$ if $\left({}^{\delta_{\mu\nu}}/_{\rho}\right) = -1$ or $\delta_{\mu\nu} = 0$, & $\varrho_\rho^*(\mu\nu) = 1$ if $\left({}^{\delta_{\mu\nu}}/_{\rho}\right) = 1$ or $\delta_{\mu\nu} = 1$, whence $|\varrho(\mu) - \varrho(\nu)| \equiv \delta_{\mu\nu}\ (mod\ \rho)$, $0 \le \delta_{\mu\nu} < \rho$, fulfills the condition $|e_{\varrho_\rho^*}(0) - e_{\varrho_\rho^*}(1)| \le 1$, whence $e_{\varrho_\rho^*}(i)$ remains the count of labelled edges i $(i = 0, 1)$. The analysis of Legendre absolute difference cordial labelling modulo ρ for certain networks, such as Y-tree graph, F- tree graph, Triangular snake graph and Trianglar book graph.**

Keywords: **Legendre absolute difference cordial Y-tree, F- tree graph, Triangular snake graph and Trianglar book graph.**

I. INTRODUCTION

Purely finite, fundamental, and connected graphs are accounted for consideration in the present analysis. A plot $G = (V, E)$ is made up of an edge set E & a vertex set V, while $|E|$ as well as $|V|$ stand for the graph's size and order, respectively. In the crucial field of graph labelling, vertices, edges, or both may be given numbers under specific circumstances. It possesses multiple advantages in communication platforms including coding theory. Quadratic congruences resembling $x^2 \equiv \alpha\ (mod\ \rho)$, at which ρ constitutes an odd prime, are of paramount significance in number theory. The Adrien-Marie Legendre symbol $\left({}^{\alpha}/_{\rho}\right)$ determines whether the congruence has a solution. If a solution exists, $\left({}^{\alpha}/_{\rho}\right) = 1$; otherwise, $\left({}^{\alpha}/_{\rho}\right) = -1$. The Legendre symbol is widely used in cryptography, computer science, and engineering [8]. In 1987, I. Cahit [7] introduced cordial labeling, a weaker form of graceful and harmonious labeling. This concept later inspired several new labeling methods, including total product cordial labeling [10]. Lucas divisor cordial labeling [12]. Some cordial labeling techniques are also applied in areas related to blood circulation and human anatomy [4],[5]. In 2025 Jason D. Andoyo and Ricky F. Rulete [9] introduced Legendre cordial labeling motivated by this work, we introduce a new graph labeling method called Legendre absolute cordial labeling [1]. By adding the Legendre sign and examining its structure of mathematics as well as practical uses, this innovative, cordial labelling expands on conventional graph labelling.

In this paper we investigate the Legendre absolute difference cordial labeling modulo ρ, which combined graph labeling techniques with the properties of quadratic residues and quadratic nonresidues modulo an odd prime ρ. We examine several tree-related graphs, including the Y-tree graph, F-tree graph, triangular book graph, triangular snake graph and banana tree graph. The existence of this labeling is established under suitable condition on ρ based on the balance between the include edge labels 0 and 1.

II. PRELIMINARIES

Definition: 2.1 Y- tree graph has $n+1$ vertices and n edges and it is denoted by Y_n is obtained from a path P_n by attaching a pendant vertex of the n^{th} vertex of P_n.

Definition: 2.2 F- tree graph has $n+2$ vertices and $n+1$ edges and it is denoted by FT_n is obtained from a path P_n by attaching exactly pendant vertex of the 1^{th} and second vertex of P_n.

Definition: 2.3 The triangular book graph $B_{3,n}$ for $n \geq 1$ is a planer undirected graph having $n+2$ vertices and $2n+1$ edges constructed by n triangles sharing a common edge.

Definition: 2.4 The triangular snake graph T_n is obtained from a path $u_1, u_2, u_3, \dots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for $1 \leq i \leq n-1$.

Definition: 2.5 A banana tree graph, denoted by $B_{n,k}$ is obtained by joining one vertex of each of n copies of the star graph $K_{1,k}$to a single new vertex.

Definition: 2.6 Letting k remain an integer deemed relatively prime with ρ, as well as have ρ represent an odd prime. If there exist a solution to the quadratic congruence $x^2 \equiv k \ (mod\ \rho)$, yields k defines a quadratic residue for ρ. If not, k corresponds to a quadratic nonresidue to ρ. If k constitutes a quadratic residue for ρ, then Legendre symbol (k/ρ) is expressed via $(k/\rho) = 1$; when not, $(k/\rho) = -1$.

Theorem 2.7 [8] Consider ρ as odd prime. So ρ contain $\frac{\rho-1}{2}$ quadratic residues as well as $\frac{\rho-1}{2}$ quadratic nonresidues belong to $\{1,2,3,\dots,\rho-1\}$.

III. MAIN RESULT

Definition:3.1 Let $G=(V,E)$, a connected, a bijection $\varrho: V(G) \to \{1,2,..,|V|\}$ is referred to as a Legendre absolute difference cordial labeling modulo ρ (LADCLMρ), with ρ denotes an odd prime, when the induced map $\varrho_\rho^*: E(G) \to \{0,1\}$ characteristic with $\varrho_\rho^*(\mu\nu) = 0$ whence $\left({}^{\delta_{\mu\nu}}/_{\rho}\right) = -1$ or $\delta_{\mu\nu} = 0$, also $\varrho_\rho^*(\mu\nu) = 1$ if $\left({}^{\delta_{\mu\nu}}/_{\rho}\right) = 1$ or $\delta_{\mu\nu} = 1$, while $|\varrho(\mu) - \varrho(\nu)| \equiv \delta_{\mu\nu} \ (mod\ \rho)$, $0 \leq \delta_{\mu\nu} < \rho$, proves the constraints $|e_{\varrho_\rho^*}(0) - e_{\varrho_\rho^*}(1)| \leq 1$, whereas $e_{\varrho_\rho^*}(i)$ indicates how many edges have been given the identity i $(i = 0,1)$. A Legendre absolute difference cordial graph modulo ρ (LADCGMρ) is a graph that allows such a labeling.

Theorem: 3.2 Y- tree graph Y_ρ is a Legendre absolute difference cordial.

Proof: Let $V(Y_\rho) = \{v_1, v_2, v_3, \dots, v_{\rho+1}\}$ and construct a bijective mapping $\varrho: V(Y_\rho) \to \{1,2,\dots,\rho+1\}$ by

$$\varrho(v_{2i-1}) = i \qquad \forall\, 1 \leq i \leq \frac{\rho+1}{2},$$

$$\varrho(v_{2i}) = \rho + 1 - i \qquad \forall\, 1 \leq i \leq \frac{\rho-1}{2}$$

$$\varrho(v_\rho) = \rho + 1$$

Then for every edge $v_i v_{i+1}, v_{\rho-1} v_{\rho+1} \in E(Y_\rho)$,

$|\varrho(v_i) - \varrho(v_{i+1})| \equiv \rho - i \ (mod\ \rho) \quad \forall 1 \leq i \leq \rho - 1$

Based on this labeling, let $\alpha = \{\delta: |\varrho(u_i) - \varrho(u_{i+1})| \equiv \delta (mod\ \rho), 0 < \delta < \rho, i = 1,2,3,\dots,\rho-1\}$. By the Theorem 2.7, the set α contains a same count of quadratic residues & quadratic nonresidues modulo ρ, namely $\frac{\rho-1}{2}$ of each. Also $\varrho(v_{\rho-1} v_{\rho+1}) = \frac{\rho-1}{2}$. $Therefore, e_{\varrho_\rho^*}(0) = \frac{\rho+1}{2}, e_{\varrho_\rho^*}(1) = \frac{\rho-1}{2}$. Hence, $|e_{\varrho_\rho^*}(0) - e_{\varrho_\rho^*}(1)| \leq 1$. Thus, Y_ρ admits a Legendre absolute difference cordial graph modulo ρ.

Example 3.3 Figure 1 illustrates the LADCLMρ for Y_ρ, with the specific case $\rho = 7$ shown as an example

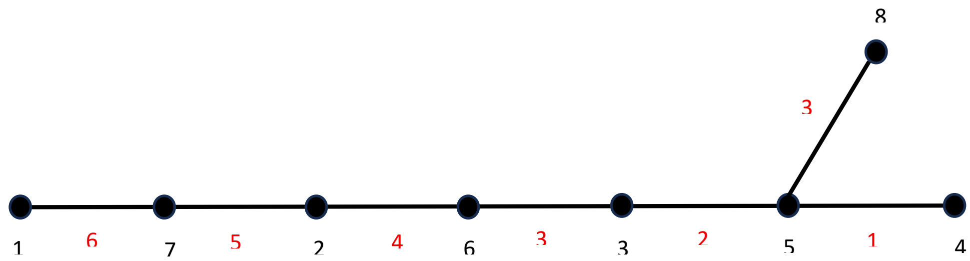

Fig. 1. LADCLM7 of Y_7

Theorem: 3.4 The F- tree graph FT_ρ admits a Legendre absolute difference cordial whenever $\rho \leq 7$.

Proof: Let $V(FT_\rho) = \{u, v, v_1, v_2, v_3, \ldots, v_\rho\}$ and construct a bijective mapping $\varrho: V(FT_\rho) \rightarrow \{1,2, \ldots, \rho + 2\}$ by

$\varrho(v_{2i-1}) = i \qquad \forall\, 1 \leq i \leq \frac{\rho+1}{2}$,

$$\varrho(v_{2i}) = \rho + 1 - i \quad \forall\, 1 \leq i \leq \frac{\rho - 1}{2}$$

$$\varrho(u) = \rho + 1$$

$$\varrho(v) = \rho + 2$$

Then for every edge $v_i v_{i+1}, uv_1, vv_2 \in E(FT_\rho)$,

$|\varrho(v_i) - \varrho(v_{i+1})| \equiv \rho - i \ (mod\ \rho) \quad \forall 1 \leq i \leq \rho - 1$

$|\varrho(u) - \varrho(v_1)| \equiv 0 \qquad |\varrho(v) - \varrho(v_2)| \equiv 2$

Based on this labeling, let $\alpha = \{\delta: |\varrho(v_i) - \varrho(v_{i+1})| \equiv \delta (mod\ \rho), 0 < \delta < \rho, i = 1,2,3, \ldots, \rho - 1\}$.By the Theorem 2.7, the set α contains a same count of quadratic residues & quadratic nonresidues modulo ρ, namely $\frac{\rho-1}{2}$ of each. Also $\varrho(uv_1) = 0$ and $\varrho(vv_2) = 2$. Therefore, $e_{\varrho_\rho^*}(0) = \frac{\rho+1}{2}, e_{\varrho_\rho^*}(1) = \frac{\rho+1}{2}$. Hence, $|e_{\varrho_\rho^*}(0) - e_{\varrho_\rho^*}(1)| \leq 1$. Suppose if we take $\rho > 7$ however apply same labeling condition it is satisfies the legendre condition but fails to satisfy the cordial condition. Therefore FT_ρ admits a Legendre absolute difference cordial whenever $\rho \leq 7$.

Example 3.5 Figure 2 illustrates the LADCLMρ for FT_ρ, with the specific case $\rho = 7$ shown as an example.

Fig. 2. LADCLM7 of FP_7

Remark: 3.6 F- tree graph FT_ρ is not a Legendre absolute difference cordial for every odd prime $\rho > 7$.

Theorem: 3.7 The Triangular book graph $TB_{3,\rho}$ admits a legendre absolute difference cordial graph whenever $\rho \leq 5$.

Proof: Let $V(TB_{3,\rho}) = \{v_1, v_2, v_3, \ldots, v_{\rho+2}\}$ and construct a bijective mapping $\varrho: V(TB_{3,\rho}) \rightarrow \{1,2, \ldots, \rho + 2\}$ by

$$\varrho(v_1) = 1$$

$$\varrho(v_2) = 3$$

$2\varrho(v_3) = 2$

$$\varrho(v_i) = 3 + i \quad \forall\, 1 \le i \le \rho - 3$$

Then for every edge $v_1v_2, v_1v_i, v_iv_3 \in E(TB_{3,\rho})$,

$|\varrho(v_1) - \varrho(v_i)| \equiv i - 1 \ (mod\ \rho) \quad \forall 4 \le i \le \rho$

$|\varrho(v_3) - \varrho(v_j)| \equiv i \ \ (mod\ p) \ \forall 1 \le i \le \rho - 3, \quad \forall 4 \le j \le \rho$

Based on this labeling, let $\alpha = \{\delta: |\varrho(v_1) - \varrho(v_i)| \equiv \delta(mod\ \rho), 0 < \delta < \rho, i = 4, \dots, \rho\}$. $\beta = \{\delta: |\varrho(v_3) - \varrho(v_i)| \equiv \delta(mod\ \rho),\ 0 \le \delta < 1, i = 1,2\}$By the Theorem 2.7, the set $\alpha \cup \beta$ contains a same count of quadratic residues & quadratic nonresidues modulo ρ, namely $\frac{\rho-1}{2}$ of each. Also $\varrho(v_3v_{\rho-1}) = 0$ and $\varrho(v_1v_3) = 1$. Therefore, $e_{\varrho_\rho^*}(0) = \rho + 1, e_{\varrho_\rho^*}(1) = \rho$. Hence, $|e_{\varrho_\rho^*}(0) - e_{\varrho_\rho^*}(1)| \le 1$. Suppose if we take $\rho > 5$ however apply same labeling condition it is satisfies the legendre condition but fails to satisfy the cordial condition. Therefore $TB_{3,\rho}$ admits a Legendre absolute difference cordial whenever $\rho \le 5$.

Example 3.8 Figure 3 illustrates the LADCLMρ for $TB_{3,\rho}$, with the specific case $\rho = 3$ shown as an example.

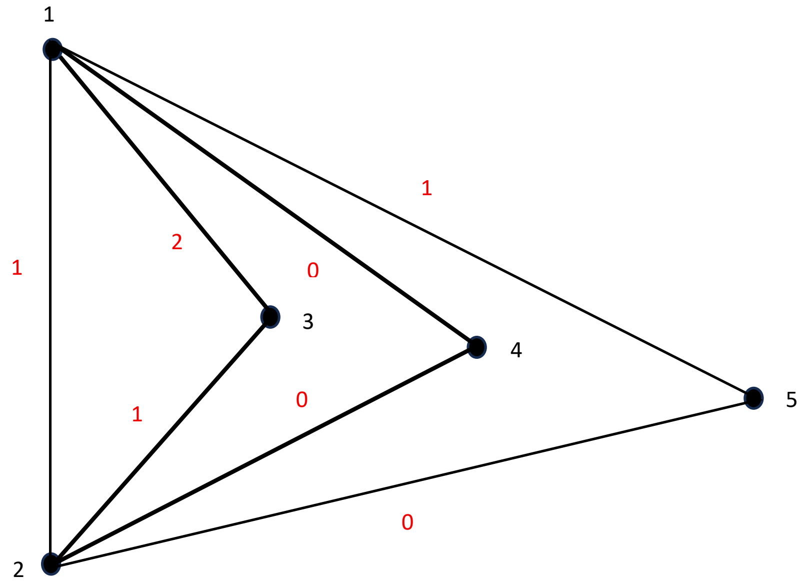

Fig. 3. LADCLM3 of $TB_{3,3}$

Remark: 3.9 The Triangular book graph $TB_{3,\rho}$ is not a Legendre absolute difference cordial for every odd prime $\rho > 5$.

Theorem: 3.10 The Triangular snake graph T_ρ admits a legendre absolute difference cordial graph whenever $\rho \le 5$.

Proof: Let $V(T_\rho) = \{u_1, u_2, u_3, \dots, u_\rho, v_1, v_2, v_3, \dots, v_{\rho-1}\}$ and construct a bijective mapping $\varrho: V(T_\rho) \to \{1,2, \dots, 2\rho - 1\}$ by

$\varrho(u_{2i-1}) = i \qquad \forall\, 1 \le i \le \frac{\rho+1}{2}$,

$$\varrho(u_{2i}) = \rho + 1 - i \qquad \forall\, 1 \le i \le \frac{\rho - 1}{2}$$

$$\varrho(v_i) = \rho + i \qquad \forall\, 1 \le i \le \rho - 1$$

Then for every edge $v_1v_2, v_1v_i, v_iv_3 \in E(TB_{3,\rho})$,

$|\varrho(u_i) - \varrho(u_{i+1})| \equiv \rho - i \ (mod\ \rho) \ \ \forall 1 \le i \le \rho - 1$

$|\varrho(u_i) - \varrho(v_i)| \equiv \rho - 2(i + 1) \ \ (mod\ \rho) \ \forall 1 \le i \le \frac{\rho+1}{2}$,

$|\varrho(u_i) - \varrho(v_i)| \equiv 2 - i \pmod \rho \ \forall 1 \leq i \leq \frac{\rho-1}{2}$

Based on this labeling, let $\alpha = \{\delta: |\varrho(u_i) - \varrho(u_{i+1})| \equiv \delta (mod\ \rho), 0 < \delta < \rho, i = 1, \dots, \rho - 1\}$. $\beta = \{\delta: |\varrho(u_i) - \varrho(v_i)| \equiv \delta(mod\ \rho),\ 0 \leq \delta < 1, i = 1, \dots, \rho - 1\}$By the Theorem 2.7, the set $\alpha \cup \beta$ contains a same count of quadratic residues & quadratic nonresidues modulo ρ, namely $\frac{\rho-1}{2}$ of each. Also $\varrho(v_{\rho-1}v_{\rho-1}) = 0$. Therefore, $e_{\varrho_\rho^*}(0) = \rho + 1, e_{\varrho_\rho^*}(1) = \rho + 1$. Hence, $|e_{\varrho_\rho^*}(0) - e_{\varrho_\rho^*}(1)| \leq 1$. Suppose if we take $\rho > 5$ however apply same labeling condition it is satisfies the legendre condition but fails to satisfy the cordial condition $\left|e_{\varrho_\rho^*}(0) - e_{\varrho_\rho^*}(1)\right| > 1$. Therefore $TB_{3,\rho}$ admits a Legendre absolute difference cordial whenever $\rho \leq 5$.

Example 3.11 Figure 4 illustrates the LADCLMρ for T_ρ, with the specific case $\rho = 5$ shown as an example.

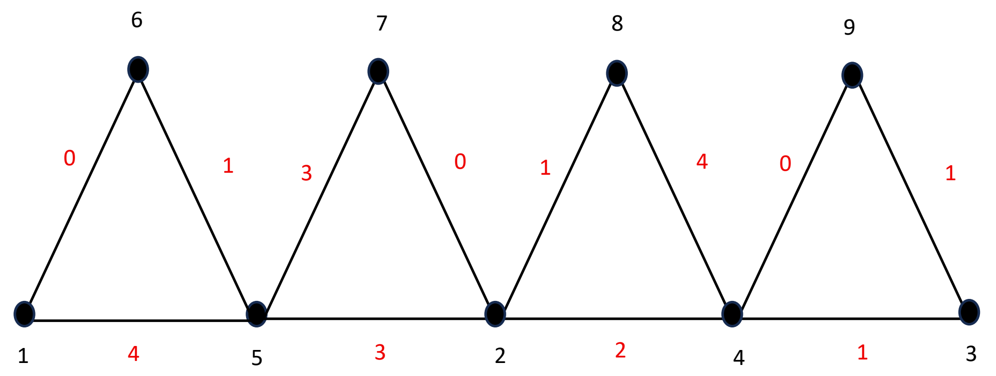

Fig. 4. LADCLM5 of T_5

Remark: 3.12 The Triangular snake graph T_ρ is not a Legendre absolute difference cordial for every odd prime $\rho > 5$.

Theorem: 3.13 The Banana tree graph $BT_{\rho,\rho}$ admits a legendre absolute difference cordial graph whenever $\rho = 3$.

Proof: Let $V(BT_{\rho,\rho}) = \{u, u_1, u_2, u_3, \dots, u_\rho, v_1, v_2, v_3, \dots, v_{2\rho}\}$ and construct a bijective mapping $\varrho: V(BT_{\rho,\rho}) \to \{1,2, \dots, 3\rho + 1\}$ by

$\varrho(u_i) = i + 1 \qquad \forall\ 1 \leq i \leq \rho,$

$$\varrho(v_i) = \rho + 1 + i \qquad \forall\ 1 \leq i \leq \rho$$

$$\varrho(v_{4i}) = 12 - 2i \qquad \forall\ 1 \leq i \leq \rho - 1$$

$$\varrho(u_{\rho-1}) = 3\rho$$

Then for every edge $v_i v_{i+1} \in E(BT_{\rho,\rho})$,

$|\varrho(u) - \varrho(u_i)| \equiv i \pmod \rho \ \ \forall 1 \leq i \leq \rho$

$|\varrho(u_1) - \varrho(u_i)| \equiv i - 1 \pmod \rho \ \forall 1 \leq i \leq 2,$

$|\varrho(u_2) - \varrho(u_i)| \equiv 1 \pmod \rho \ \forall 3 \leq i \leq 4$

$|\varrho(u_3) - \varrho(u_i)| \equiv j - 1 \pmod \rho \ \forall 5 \leq i \leq 6, j = 1,2$

Based on this labeling, let $\alpha = \{\delta: |\varrho(u_i) - \varrho(u_{i+1})| \equiv \delta (mod\ \rho), 0 < \delta < \rho, i = 1, \dots, \rho - 1\}$. $\beta = \{\delta: |\varrho(u_i) - \varrho(v_i)| \equiv \delta(mod\ \rho),\ 0 \leq \delta < 1, i = 1, \dots, \rho - 1\}$By the Theorem 2.7, the set $\alpha \cup \beta$ contains a same count of quadratic residues & quadratic nonresidues modulo ρ, namely $\frac{\rho-1}{2}$ of each. Therefore, $e_{\varrho_\rho^*}(0) = \rho + 1, e_{\varrho_\rho^*}(1) = \rho + 2$. Hence, $|e_{\varrho_\rho^*}(0) - e_{\varrho_\rho^*}(1)| \leq 1$. Suppose if we take $\rho \neq 3$ however apply same labeling condition it is satisfies the legendre condition but fails to satisfy the cordial condition $\left|e_{\varrho_\rho^*}(0) - e_{\varrho_\rho^*}(1)\right| > 1$. Therefore $BT_{\rho,\rho}$ admits a Legendre absolute difference cordial whenever $\rho \neq 3$.

Example 3.14 Figure 5 illustrates the LADCLMρ for $BT_{\rho,\rho}$ with the specific case $\rho = 3$ shown as an example.

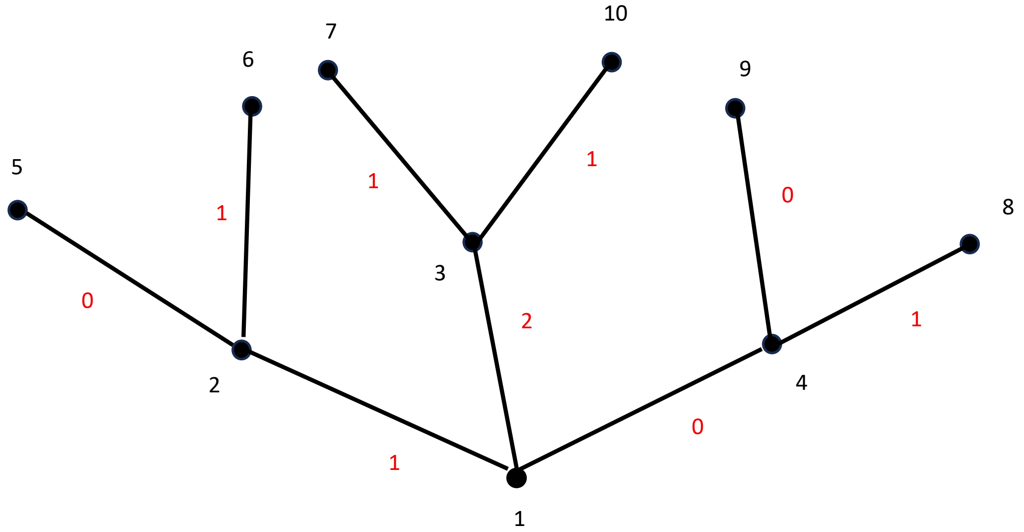

Fig. 5. LADCLM3 of $BT_{3,3}$

Remark: 3.12 The Banana tree graph is not a Legendre absolute difference cordial for every odd prime $\rho \neq 3$.

IV. CONCLUSION

In this study underscores the significance of **Legendre Absolute Difference Cordial Labeling Modulo** ρ for several tree-related and graph families. The results show that the Y-tree graph admits a Legendre absolute difference cordial labeling for every odd prime. The F-tree graph admits the labeling when $\rho \leq 7$, whereas the triangular book graph and triangular snake graph admit it when $\rho \leq 5$. Furthermore, the banana tree graph admits the labeling only for $\rho = 5$. These results demonstrate the influence of quadratic residues and nonresidues on the cordiality condition.

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